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Theory of disordered contacts in high magnetic fields: weak disorder

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Abstract. We consider a model of a weakly disordered wire connected to a clean sample. The disordered wire acts as an imperfect contact, coupled in varying degrees to different out going and in coming states in the sample. We consider here a strong magnetic field where the states are edge states, and scattering between the different edge states is weak. We derive the distribution between edge states of electrons emerging from an infinitely long, disordered current lead, and the sensitivity of an infinitely long voltage lead to electrons in different edge states, in terms of the ratio of the backscattering rate to the scattering rate between the inner and outer edge states on one side of the lead. Corrections for finite lead lengths are exponentially small so long as the lead is long compared to the backscattering length. We discuss how the results relate to recent experiments.

1. Introduction

We consider in this paper a model of a weakly disordered wire in a magnetic field. The wire is modelled in terms of the scattering between the edge states in the wire, and we consider particularly how such a wire behaves as a contact, or lead. The analysis is appropriate for weak disorder, where edge states are well defined. We consider in a separate paper the case of strong disorder.

In a recent paper Geim *et al* [1] showed experimentally that a disordered contact gives a non-equilibrium, strongly temperature dependent, occupation of the different edge states that exist in a wire in a large magnetic field.

From Büttiker [2] onwards it has been known that, in general, contacts are not ideal, that is they do not populate all modes equally. The general properties of such a disordered contact have been considered by Komiyama and Hirai [3]. In this paper we consider the properties of a long disordered wire in a strong magnetic field, and show how such a wire behaves as a non-ideal contact.

In the first part of the paper we discuss how the non-equilibrium distribution arises in terms of the difference between the backscattering rate and the scattering between adjacent edge states. We show how, in a coupled edge state model similar to that used by McEuen *et al* [4], a long wire populates the edge states in a manner which is independent of the length of the wire. We find for weak interedge scattering a square root dependence of the relative occupation of the outgoing edge states upon the ratio between interedge state scattering on one side of the wire and backscattering. We gain some insight into the behaviour of the model, and the nature of the finite-length corrections, from considering a reduced model

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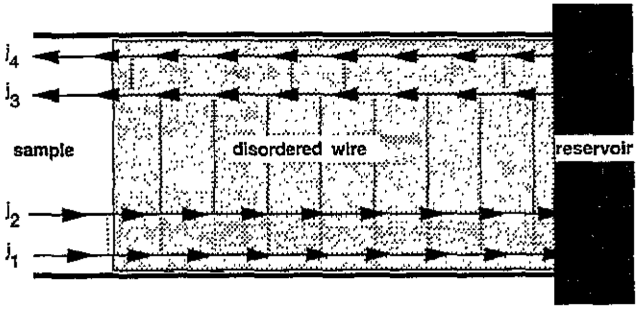

Figure 1. We consider a disordered wire, with scattering between edge states on the one side of the wire, and backscattering between the two sides. A reservoir at one end of the wire absorbs electrons from the right going edge states, and equally populates the left going edge states. At the other end the disordered wire is connected to a sample. We are interested in the response of the chemical potential of the reservoir to the incoming currents j_1 and j_2 and in the relative size of the outgoing currents j_3 and j_4.

where we ask how many electrons in the innermost edge state are reflected back from the sample without ever being scattered to one of the outer edge states. Finally we discuss the consequences of this picture for the temperature dependence.

2. Coupled edge state model

We consider the system shown in figure 1. The system consists of a lead connecting an ideal reservoir to a sample, the lead being a disordered wire in a sufficiently large magnetic field that the electrons are in edge states. It is known that equilibration between edge states in the quantum Hall regime is impeded by the spatial separation, and by the momentum difference, between the different edge states [5, 6, 7]. This is particularly so for equilibration between the innermost edge state and the outer edge states. We therefore assume that all the edge states, except the innermost, are in mutual equilibrium, and describe the system by the scattering rates between different edge states shown in the figure. Note that we assume that the only transfers are between the inner and the outer edge states, and that there is no backscattering between the two outer edge states in opposite directions: in other words, there is no direct transmission between the outer edge states on opposite sides of the sample, although electrons can pass between these states by a sequence of transfers. We make no assumption about the nature of the scattering between the edge states: so long as there is sufficient inelastic scattering that the currents in each state can be described by a single chemical potential (or equivalently by the current in the channel) the model is adequate to describe the occupation of the different states.

To analyse the system, we assume that the scattering lengths are long compared with the width of the system, and decompose the system into short lengths, with currents as shown in figure 2. Throughout this paper we mean by current the excess current associated with some applied potential: there is of course a circulating current from electrons below the Fermi energy. With this understanding, the current in an edge state and the chemical potential of an edge state measured relative to some energy are equivalent:

$$j = \frac{e^2}{2h}\mu. \tag{1}$$

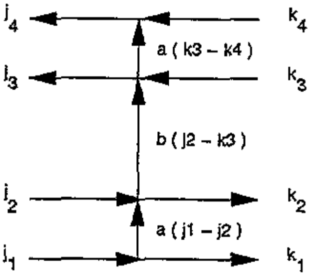

Figure 2. The wire is broken into short sections like the one shown. Interedge state scattering with rate a and backscattering with rate b lead to net currents between the edge states if their chemical potentials differ.

The discussion in this paper is framed in terms of the currents in each edge state; the translation to chemical potentials where desired is trivial. For simplicity we consider the case with just two edge states. So long as the outer edge states are in mutual equilibrium, the extension to the case of several edge states is likewise trivial.

For one section we have from current conservation that

$$\begin{aligned} k_1 &= j_1 - a(j_2 - j_1) \\ k_2 &= j_1 + a(j_2 - j_1) + b(j_3 - k_2) \\ j_3 &= k_3 + a(k_4 - k_3) - b(j_3 - k_2) \\ j_4 &= k_4 - a(k_4 - k_3) \end{aligned} \tag{2}$$

where the currents k_1, k_2, ... are defined with reference to figure 2. The coefficient a represents the interedge scattering, and the coefficient b the backscattering. (We make two comments. One could equivalently write down a set of differential equations for the evolution of the currents as Komiyama and Hirai did for the equilibrium between edge states on one side of a quantum Hall bar [3], when one would be interested in the limit $a \to 0$, $b \to 0$, $a/b = r$. Also, terms such as $a(k_2 - k_1)$ could be included in the equations, but make no material difference, and are irrelevant in the limit where the scattering rate in the given length is small.)

We can rearrange (2) to find

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} = \mathbf{T} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix} \tag{3}$$

where

$$\mathbf{T} = \begin{pmatrix} 1 - \frac{a}{2} & \frac{a}{2} & 0 & 0 \\ \frac{a}{2} & \frac{8-12a+4a^2-8b+8ab-a^2b}{2(4-4a-2b+ab)} & \frac{(2-a)b}{4-4a-2b+ab} & -\frac{ab}{4-4a-2b+ab} \\ 0 & \frac{(-2+a)b}{4-4a-2b+ab} & \frac{2(2-a)}{4-4a-2b+ab} & \frac{-2a}{4-4a-2b+ab} \\ 0 & \frac{ab}{4-4a-2b+ab} & \frac{-2a}{4-4a-2b+ab} & \frac{-2(-2+a+b)}{4-4a-2b+ab} \end{pmatrix} \tag{4}$$

which gives the evolution of the current distribution along the wire, and after N sections,

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} = \mathbf{T}^{\mathrm{N}} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}. \tag{5}$$

We now consider the relevant boundary conditions in a long lead. We assume that we have at one end of the lead an ideal reservoir, with chemical potential μ which populates equally all the edge states leaving it, so that $k_1 = k_2 = (e^2/2h)\mu$. (Note that whether or not the reservoir is actually ideal is irrelevant to the relative occupations of different modes by the wire, since the scattering in the wire will lead to the same eventual relative occupation of the modes, regardless of how the electrons are injected into the wire from the reservoir.) In general we then need to specify the currents incident on the lead from the sample, j_1 and j_2, and either the net current in the wire, or the chemical potential of the reservoir. We will consider two special cases which we refer to as a voltage lead, and a current lead. The problem is linear, so the general case can be solved as a linear superposition of these two cases.

The first case is a voltage lead, that is a lead with no net current flowing, the reservoir having a suitable chemical potential, which gives the measured voltage. The other boundary conditions are the incoming currents j_1, j_2 and the requirement that no net current flow: $k_1 + k_2 + k_3 + k_4 = 0$.

The second case is a current lead, that is a lead in which there is a net current flowing in the lead, and where we are interested in the current flowing out of the lead and into the sample. As boundary conditions we impose $j_1 = j_2 = 0$, that is no electrons are reflected back from the sample into the lead, and we choose a chemical potential μ. If there is any current flowing into the lead, so that j_1 or j_2 is not zero, it will be partially reflected, and add to the current flowing out of the lead.

2.1. Limiting behaviour

Komiyama and Hirai have shown quite generally [3] that the response, μ, of a voltage lead with several incoming modes is a weighted average of the chemical potentials of the different modes. In our case the modes are the two edge states, so that

$$(e^2/2h)\mu = \frac{1}{(1+\alpha)} j_1 + \frac{\alpha}{(1+\alpha)} j_2 \tag{6}$$

where α is the ratio between the sensitivity of the chemical potential of the reservoir to the current in the inner edge state, $\partial\mu/\partial j_2$, and the sensitivity of the chemical potential of the reservoir to the current in the outer edge state $\partial\mu/\partial j_1$. If there is no backscattering in the system, so that all electrons entering the disordered region reach the reservoir, then $\alpha = 1$. If the backscattering is strong, compared with the equilibration between the inner and outer edge states, then $\alpha = 0$, and the reservoir is only sensitive to electrons in the outer edge states: any electron coming into the lead in the inner edge state is backscattered without influencing the chemical potential of the reservoir.

Consider now a current lead. The lead has some resistance, dependent on the backscattering. More germane to our purpose, the channels going out from the disordered region into the clean wire, j_3 and j_4, are populated differently. In the absence of backscattering j_3 and j_4 are equal. In the limit of strong backscattering no current emerges in the outer edge state. (The general analysis of Komiyama and Hirai [3] shows that the response of a voltage lead to the different edge states is a weighted average of their potentials, and that the ratio of currents in the outgoing channels of a current lead, j_3/j_4, is the same as the ratio of the sensitivities of the reservoir's chemical potential to the edge states for the voltage lead, $(\partial\mu/\partial j_2)\,/\,(\partial\mu/\partial j_1))$.)

The relative occupations in this high magnetic field case are interestingly opposite to what has been observed in zero and low magnetic fields by Blaikie *et al* [8]. In their case, the cyclotron radius was larger than, or comparable to, the sample width, and modes at the edge were preferentially scattered.

2.2. *Numerical behaviour*

We have not been able to solve (5) analytically for an infinite wire since the matrix **T** does not have a complete set of eigenvectors. We present here therefore numerical results showing the behaviour of current and voltage leads as a function of a/b. In the next section we use the physical insight derived from the numerical solution to solve the problem.

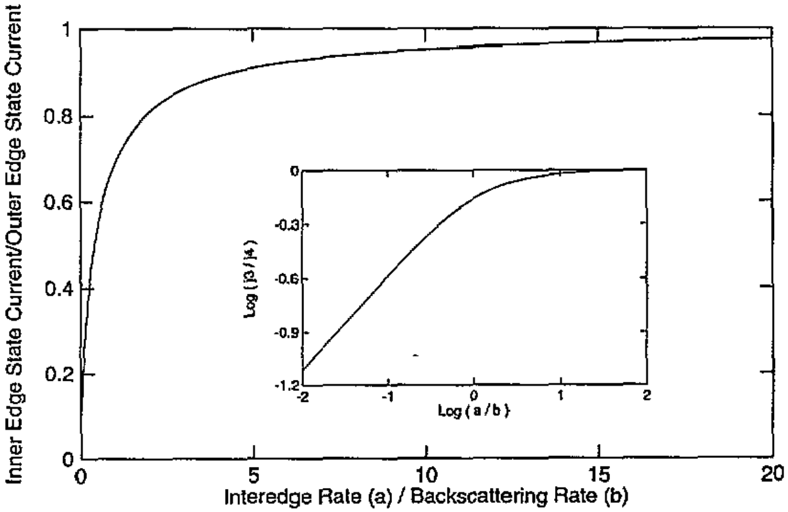

Figure 3. Relative occupation of edge states by a current lead as a function of the ratio of scattering rates. Inset is a log–log plot of the same data showing the square root behaviour for weak interedge scattering.

Figure 3 shows the relative occupation of the inner edge state and the outer edge state emerging from the lead into the sample. (As we remark above, this is equivalent to the ratio of the sensitivity of the potential of the reservoir of a voltage lead to the inner and to the outer edge states incident on the lead from the sample.) We find that, for small a, the occupation of the inner edge state is proportional to $\sqrt{a/b}$, a feature which we explain below. For large a all the outgoing edge states are equally occupied, as expected from the discussion above.

Figure 4 shows how the currents in the edge states vary along the length of a voltage lead. Close to the interface between the sample and the lead the potentials deviate towards the external values, going into the lead from the sample the chemical potentials decay exponentially towards an equilibrium value. The matrix **T** has eigenvalues 1, 1, λ, λ^{-1}. (The existence of a unit eigenvalue is obvious: equal chemical potentials in each mode forms a uniform solution. Since $\mathbf{T}^{-1}$ describes the evolution of the chemical potentials in the opposite direction to **T** the eigenvalues must be in reciprocal pairs.) As figure 4 shows, the currents in the voltage lead are composed of a constant part plus a part decaying into the lead away from the sample, which is described by the eigenvector of **T** with an eigenvalue smaller than unity. We use this observation in the next section to solve the problem.

For a current lead, where there is dissipation along the length of the wire the situation is a little more complicated. The currents are shown in figure 5. In the interior of the lead, there is a constant current difference between the modes. Unlike the voltage lead the currents are forced to deviate from this pattern both at the entrance, and at the reservoir.

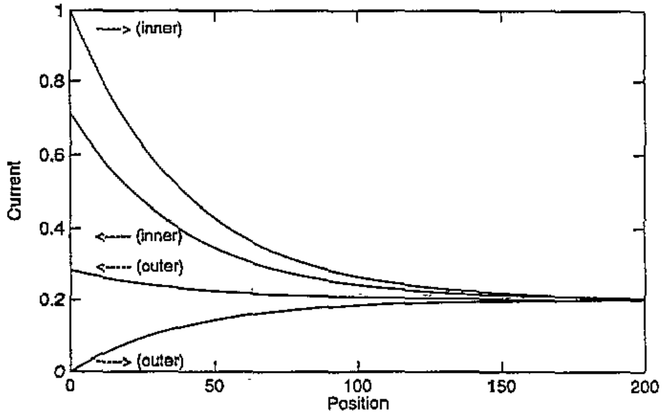

Figure 4. Currents in the different edge states in a voltage lead as a function of position away from the sample, in units of the slices the wire is divided into. $a = 0.01$, and $b = 0.1$. Unit current is injected from the sample in the inner of the two incoming edge states (j_2).

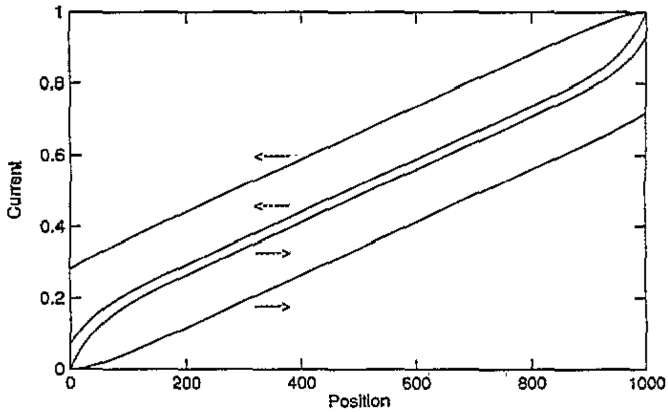

Figure 5. Currents in the different edge states in a current lead as a function of position away from the sample, in units of the slices the wire is divided into. $a = 0.01$, and $b = 0.1$. The outer pair of lines is the outer edge states.

Well away from the interfaces between the lead and the sample, and between the lead and the reservoir, there is a constant voltage drop from one section to the next, so that the currents are described by the equation

$$\mathbf{T}\begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} = \begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} + \delta\mu \tag{7}$$

where $\delta\mu$ is the voltage drop across one of the sections of the wire. We can solve to find the currents

$$\begin{pmatrix} j_1 \\ j_2 \\ j_3 \\ j_4 \end{pmatrix} = \begin{pmatrix} 1 - 4/a - 4/b \\ 1 - 2/a - 4/b \\ -2/a \\ 0 \end{pmatrix} \delta\mu + c \tag{8}$$

where c is an arbitrary constant, and the net current is

$$2\,(1 - 4/b - 2/a)\delta\mu. \tag{9}$$

Note that as a or b goes to zero, so that either there is no backscattering, or the outer edge states are isolated from the backscattering, the current for a given potential drop along the wire diverges, that is, the resistance goes to zero, as it should in the quantum Hall regime.

2.3. Solution

We now use the fact that the currents at the end of a voltage lead can be decomposed into the uniform solution and the eigenvector of currents decaying into the sample to find the ratio of the potential response to the two input modes. Let the uniform solution be written $(1 \quad 1 \quad 1 \quad 1)$, and the decaying solution be written $(x_1 \quad x_2 \quad x_3 \quad x_4)$. Solving

$$\mu\,(1 \quad 1 \quad 1 \quad 1) + \beta\,(x_1 \quad x_2 \quad x_3 \quad x_4) = (j_1 \quad j_2 \quad j_3 \quad j_4) \tag{10}$$

for μ and β with j_1 and j_2 fixed, we see that the ratio of sensitivities is

$$\frac{\partial\mu/\partial j_2}{\partial\mu/\partial j_1} = -\frac{x_1}{x_2}. \tag{11}$$

The eigenvalues of $\mathbf{T}$ are 1, and

$$\begin{aligned}(8 - 8a + 4a^2 - 4b + 4ab - a^2 b &\\ \pm \sqrt{-2+a}\sqrt{a}\sqrt{8 - 4a - 4b + ab}\sqrt{-4a - 2b + ab}) &\\ \times [2\,(4 - 4a - 2b + ab)]^{-1}. &\end{aligned} \tag{12}$$

The form of the eigenvector corresponding to the decaying mode is complicated, however, the ratio between the sensitivities is quite simple, and is given by

$$\frac{\partial\mu/\partial j_2}{\partial\mu/\partial j_1} = \frac{\sqrt{a}\,(4 - 4a - 2b + ab)}{4\sqrt{a} + \sqrt{-2+a}\sqrt{8 - 4a - 4b + ab}\sqrt{-4a - 2b + ab}} \tag{13}$$

which becomes in the limit $a = br$, $b \to 0$, that is to say, in the limit where the scattering in each slice is weak,

$$\frac{\partial\mu/\partial j_2}{\partial\mu/\partial j_1} = \frac{1}{\sqrt{4 + 2/r} - 1} \tag{14}$$

where the square root form of the sensitivity as a function of the ratio of scattering rates is evident. In the next section we present a simpler model which demonstrates how this form arises.

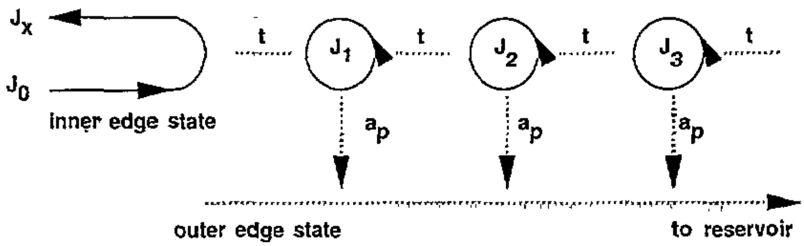

Figure 6. Pseudo-absorption model. Only the current flowing in the inner edge state is considered. Current is transmitted between adjacent sections with rate t and is absorbed (modelling transfer to the outer edge state) with rate a_p. We are interested in J_x, the amount of current reflected from the wire without being transferred into an outer edge state.

2.4. *Pseudo-absorption model*

From the point of view of experiment [1] we are interested in the limit where the backscattering in the disordered lead is relatively strong, and the disordered lead is only weakly sensitive to the inner edge state coming from the sample. As the simplest model containing the relevant physics, we consider just this inner edge state, and ask how many of the electrons which are incident in the inner edge state on the disordered lead are reflected back from the lead without being scattered into one of the outer edge states. The electrons which are scattered into the outer edge states are assumed to make contact with the reservoir. In an infinitely long lead all the other electrons are backscattered by the lead: in a finite lead some electrons will reach the reservoir directly via the inner edge states, and we consider the effect of this in the next section. We now show that this model gives the same square root dependence of the ratio of the response to the inner edge state upon the ratio between the interedge scattering and the backscattering rates as the full model.

Figure (6) shows the system we are considering. We model scattering into the outer edge states by a pseudo-absorption rate, a_p, representing the removal of electrons from the set that will be reflected back. a_p describes the same process as the interedge state scattering rate, a, in the previous sections. Electrons are transmitted from one section to the next with a rate t. We write the equations in terms of a transmission and absorption rate per unit time rather than a rate per unit length as in the previous section, since this gives the simplest set of equations, and is sufficient for understanding the behaviour qualitatively. We consider a voltage lead, so that equal currents are flowing in each direction, and we can describe the current in each section by a single variable, J_n. The steady state solution, which is what we are interested in, is then determined by

$$-a_p J_n + t(J_{n+1} + J_{n-1} - 2J_n) = 0. \tag{15}$$

There are two homogeneous solutions to the recurrence relation (15),

$$J_n = J\exp(\pm\kappa n) \tag{16}$$

where

$$\exp(\pm\kappa) = 1 + \frac{a_p}{2t} \pm \sqrt{\frac{a_p}{4t}}\sqrt{\frac{a_p}{t} + 4}. \tag{17}$$

The unabsorbed current decays into the lead, so the relevant solution has the minus sign. At the sample end of the lead we have some incoming current, J_0. The outgoing current, J_x, is then tJ_1, where

$$J_1 = J_0 \exp \kappa \tag{18}$$

plus the current, $(1-t)J_0$, that was reflected directly by the wire. The total current being absorbed is therefore

$$\begin{aligned} J_a = J_0 - J_x &= J_0 - \left((1-t)J_0 - tJ_1\right) = tJ_0\left(1 - e^{-\kappa}\right) \\ &= J_0\left(\frac{\sqrt{a_p}\sqrt{a_p + 4t} - a_p}{2}\right). \end{aligned} \tag{19}$$

In the limit where $a_p \to 0$ this becomes $J_a = t\sqrt{a_p}J_0 + O(a_p)$ and we find the square root dependence again, which we see comes from the dispersion relation.

2.5. *Finite-size effects*

For a lead which is short compared to the decay length of $L = 1/\kappa$ (or the corresponding length in the fuller model) there will be corrections to the ratio of sensitivities, since we expect that there will be some direct transmission of the inner edge state to the reservoir, thus increasing its coupling to the reservoir. Figure (7) shows how the sensitivity of a voltage reservoir to different modes changes as a function of the length of the wire. The effect of the finite length of the lead is to increase the coupling of the reservoir to the inner edge state.

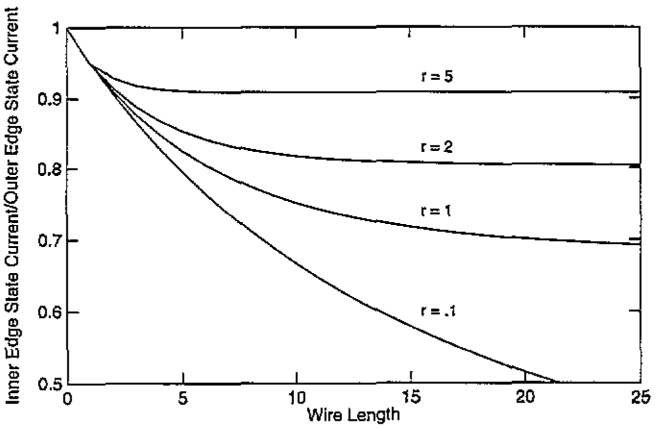

Figure 7. Relative occupation of edge states by a current lead as a function of its length in slices for different ratios $r = a/b$. b is held fixed at 0.1.

In the pseudo-absorption model we can calculate exactly the total current either absorbed in the lead or reaching the reservoir, and we find

$$J_a = tJ_0\left(1 - e^{-\kappa}\right)\frac{\left(1 + e^{-(2N-1)\kappa}\right)}{1 - e^{-2N\kappa}}. \tag{20}$$

Note that the correction is exponential in the length of the lead because of the pseudo-absorption term. For a lead of length L with no absorption, the probability of an electron reaching the far end goes as $1/L$, giving a resistance proportional to the length. In this paper we are not interested in the total resistance of the lead, but in its relative sensitivity to electrons in different edge states, and for a long wire this is a property of the end of the lead, where it joins the sample.

3. Temperature dependences

We do not attempt here a detailed discussion of the temperature dependence seen experimentally, but rather we just make some qualitative observations.

Geim *et al* [1] have observed the Shubnikov–de Haas resistance oscillations in a sample consisting of a thin, clean wire fed by disordered leads. The Shubnikov–de Haas oscillations are understood to occur by resonant tunnelling [9] between the oppositely directed inner edge states within the clean wire. The size of the resistance oscillations seen is then a measure of the population of the inner edge state by the leads: the wire is short enough that, especially in the region of the oscillations, interedge scattering within the relatively clean wire is weak. Geim *et al* found the amplitude of the oscillations to be proportional to $\exp(-\alpha T^{-1})$ close to the maximum of the Shubnikov–de Haas oscillations, and proportional to $\exp(-\alpha T^{-1/2})$ further away.

In the weak-scattering limit, that is interedge and back scattering lengths that are long compared to the width of the wire, we have seen that the occupation of the inner edge state is controlled by the ratio of the interedge scattering rate to the backscattering rate. The suppression of the occupation of the inner edge state at low temperatures can be understood in this picture if the interedge scattering rate is small compared with the backscattering rate at low temperatures. It is known [4, 5, 6] that the interedge state scattering rate is strongly temperature dependent, chiefly because of the increasing overlap between the edge states as the energy is increased [6,7]. The energy and temperature dependence of the backscattering is less well understood. Empirically [10, 11] a dependence of the form of $\exp(-\alpha T^{-1/2})$ has been found, and several theoretical explanations of this behaviour have been proposed [12, 13, 14]. To explain the experiments the interedge scattering rate must increase faster than this. However in reality, whilst some mechanisms of interedge state scattering produce an $\exp(-\alpha T^{-1/2})$ behaviour, at low temperatures the interedge state scattering rate increases linearly [6].

A full explanation of the experiments of Geim *et al* requires us to recognise that, because the experiments use the Shubnikov–de Haas oscillations in a wire with similar properties to the lead, but with less disorder, as a probe of the occupation of the different edge states, we are interested especially in the region where the inner edge state is just starting to propagate in the lead. The model described in this paper has limited validity in that case, because the backscattering length eventually becomes short compared with the width of the wire. The theories for the interedge state scattering rate also assume that the system is far enough away from the transition that the edge states are well defined.

Consistent with the picture in this paper that the relative magnitude of interedge scattering and backscattering is important, Geim *et al* find that in samples where the leads consist of clean wire (that is with the same disorder as in the bulk of the sample) the suppression of the Shubnikov–de Haas oscillations is much smaller, that is, the occupation of the inner edge state is larger. For a sample where the leads consist of a clean wires which widens rapidly so that backscattering in the wire is reduced further, the oscillations are still less suppressed.

4. Conclusions

We have presented an analysis of edge states propagating in a disordered lead, and shown how backscattering leads to a non-equilibrium occupation of the edge states, and an unequal sensitivity to different edge states incident on the lead. We have indicated qualitatively how changes in the backscattering and interedge scattering rates can lead to the experimentally observed dependence on geometry and disorder. While we have indicated how the temperature dependence of the Shubnikov–de Haas oscillations might be explained within this model of weak disorder, the necessary variations of the scattering rates seem implausible.

The model of edge states with scattering between them breaks down if the disorder, and so backscattering, is too large. Close enough to the point where an edge state is ceasing to propagate this will always be true, and we consider this regime in the following paper, where we explain qualitatively the experimental temperature dependence.

Acknowledgments

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